Time Consistent Monetary Policy with Endogenous Price Rigidity\footnote{I thank Stefania Albanesi, Roc Armenter, Larry Christiano, Wouter den Haan, Huberto Ennis, Allen Head, Nooman Rebei, Victor Rios-Rull, Andreas Schabert, Alex Wolman, and workshop participants at the Bank of Canada, UBC, UT Austin, Washington, Columbia, the 2004 Vienna Macro Workshop, the 2004 CMSG Meeting, the 2005 SED Meeting, and the 2005 NBER Summer Institute for helpful comments and discussions. I am particularly grateful to Mick Devereux, Francisco Gonzalez, and Ed Nosal for discussions and helpful advice. Research funding from the SSHRC is gratefully acknowledged. All errors are mine.}

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Abstract

In this paper I characterize time consistent equilibrium in an economy with price rigidity and an optimizing monetary authority operating under discretion. Firms have the option to increase their frequency of price change, at a cost, in response to higher inflation. Previous studies, which assume a constant degree of price rigidity across inflation regimes, find two time consistent equilibria – one with low inflation, the other with high inflation. In contrast, when price rigidity is endogenous, the high inflation equilibrium ceases to exist. Hence, time consistent equilibrium is unique. This result depends on two features of the analysis: (1) a plausible quantitative specification of the fixed cost of price change, and (2) the presence of an arbitrarily small cost of inflation that is independent of price rigidity.
1. INTRODUCTION

Central bank policy is best characterized as being set with discretion. That is, monetary policy makers do not simply implement policy plans determined in the past. So while it is crucial to characterize optimal policy under commitment, it is equally important to understand what outcomes arise when it is recognized that policy makers act with discretion. In this paper, I characterize time consistent equilibrium in a model with monetary discretion and an endogenously determined degree of price rigidity. The objective is to determine whether the model plausibly generates self-fulfilling, high inflation equilibria.

Kydland and Prescott (1977) and Barro and Gordon (1983) describe linear-quadratic economies in which the interaction between monetary discretion and a forward-looking private sector produce unique equilibrium. This equilibrium displays expected and realized inflation higher than that obtained under commitment. More recently, the issue has been studied in dynamic general equilibrium models of the monetary transmission mechanism. In these economies, equilibria are generally not unique. Expectation traps arise in which equilibria associated with expectations of low or high inflation become self-fulfilling. Hence, these models rationalize the view that the experience of the US during the 1970’s was due to a high inflation expectation trap.

Using the methods of Chari and Kehoe (1990), Chari et al. (1998) demonstrate this multiplicity in a sticky price model in which agents play trigger strategies (see also Barro and Gordon, 1983, Section IV). An important shortcoming, however, is that the play of trigger strategies admits many possible equilibria.¹ Two recent papers – Albanesi et al. (2003) and King and Wolman (2004) – study discretionary policy when reputational mechanisms are ruled out. These papers show that expectation traps remain; that is, multiplicity does not rely on folk-theorem type reasoning, but is a germane feature of monetary discretion.

The intuition can be roughly summarized as follows. Firms are monopolistic and set sticky prices. Price rigidity provides an incentive for the monetary authority to generate

¹In a highly related framework, Ireland (1997) shows that the same model that predicts expectation traps predicts the first-best, commitment solution as an equilibrium outcome as well.
unexpected inflation. Since the output of sticky price firms is demand determined, this stimulates output and reduces the monopoly distortion. Costs associated with realized inflation generate a trade-off, so that the monetary authority produces positive, but finite, inflation. Forward-looking firms account for this when setting prices. If firms coordinate expectations on low inflation occurring, they set accordingly low prices. If firms expect high inflation, they set high prices. Accommodation by the monetary authority validates private sector expectations. Hence, accommodation generates the possibility of multiple equilibria. And accommodation is precisely the hallmark of policy discretion.

A problem with this reasoning is that it relies heavily on the degree of price rigidity being exogenously given. With sticky prices, a firm’s future price is not permitted to adjust for inflation that happens between now and then. Expectations of high inflation lead firms to set high prices now, thus compelling the monetary authority to deliver on those expectations. While assuming exogenously rigid prices is fruitful for monetary business cycle analysis, it seems problematic in formulating an explanation for high inflation episodes such as the 1970’s experience. This is particularly true since the exogeneity of price rigidity is central to generating high inflation equilibria in these models.

Here, I consider an economy in which the degree of price rigidity is endogenous. The objective is to determine the robustness of the expectation trap result in such a model, absent an appeal to reputational mechanisms. In the face of high inflation, firms can choose to incur a fixed cost to increase their frequency of price change. When the degree of price rigidity is allowed to adjust, the high inflation equilibrium ceases to exist. Time consistent equilibrium is unique. This result depends on: (1) a quantitatively reasonable specification of the fixed cost of price change, and (2) the presence of an arbitrarily small welfare cost of realized inflation that is independent of rigid prices.

I show this in two steps. First, I consider a ‘simplified’ model in which realized inflation is costly only when prices are sticky, so that only feature (1) is operational. Two time

\[ \text{2}\text{Ireland (2000) shows how multiplicity in the class of models considered by Ireland (1997) and Chari et al. (1998) can be eliminated by relaxing the assumption of rational expectations. For a critical assessment of expectation traps closer in spirit to that considered here, see Barseghyan and Di Cecio (2005).} \]
consistent equilibria exist, one with low inflation, the other with high inflation. For reasonable specifications of the fixed cost of price change, the steady state of the high inflation equilibrium displays full price flexibility. With full flexibility, the cost-benefit trade-off in inflation disappears and the monetary authority is indifferent across inflation outcomes. Next, I introduce feature (2), an arbitrarily small cost of inflation that is present regardless of whether prices are sticky or flexible. This breaks the monetary authority’s indifference at full price flexibility. Thus, the high inflation equilibrium is eliminated in quantitatively relevant versions of the model, so that time consistent equilibrium is unique.

Section 2 presents the simplified model and Section 3 characterizes equilibrium for arbitrary monetary policy. Section 4 details the crucial strategic complementarity in firm’s pricing decisions that is the source of multiplicity, and how this depends on the specification of policy. Section 5 characterizes Markov perfect equilibrium in which the discretionary monetary authority maximizes private sector welfare. Section 6 analyzes Markov perfect equilibrium and Section 7 discusses the arbitrarily small perturbation of the model that eliminates the high inflation equilibrium. Section 8 concludes.

2. THE MODEL

Consider a perfect foresight, infinite horizon economy populated by: a representative final good firm; a continuum of monopolistically competitive intermediate good firms; a representative household; and a discretionary monetary authority. Sticky prices among intermediate good firms admits non-neutral effects of monetary policy.

Specifically, firms make a pricing decision every second period; half of firms do so in odd periods, the other half in even, so that pricing decisions are staggered. This friction alone generates the welfare trade-off in inflation for the monetary authority (hereafter MA). Unexpected inflation erodes the real value of sticky prices, reducing the monopoly pricing distortion. But with staggered pricing, realized inflation generates a relative price distortion, and hence misallocation of resources across firms. In making its policy choice, the MA must balance the marginal benefit of inflation, from the erosion of the monopoly distortion, with
the marginal cost, from the exacerbation of the relative price distortion. To introduce endogenous price rigidity, a firm’s pricing decision is modeled as having two dimensions: the price(s) to charge and the frequency of price change. I elaborate on this below.

Timing within a period is as follows: first, the MA chooses the growth rate of the money stock; after observing this, private sector decisions are made. At the beginning of a period, the state observed by the MA is denoted \( s \in \sigma \), which I call the MA state. To illustrate the mechanisms generating multiplicity or uniqueness as clearly as possible, reputational mechanisms are explicitly ruled out. Attention is restricted to the play of Markov strategies, so that \( s \) contains only fundamental or ‘pay-off relevant’ information. Firms make pricing decisions every second period so the oldest price being charged at any point in time is one period old. The MA’s strategy is therefore conditioned only on information inherited from the previous period. In particular, \( s = (\bar{p}, z) \), where \( \bar{p} \equiv \bar{P}_{t-1}/M_{t-1} \) is the normalized sticky price set by firms making pricing decisions in the previous period, and \( z \equiv z_{t-1} \) is the fraction of those firms choosing to set prices on a period-by-period basis. After observing \( s \), the MA chooses a gross money growth rate, \( X \). That is, if \( M_t \) is the money stock at date \( t \), \( X_t = M_t/M_{t-1} \).

Private sector decisions are made after observing \((s, X)\). Call \((s, X)\) the private sector or PS state. Among these decisions are choices for \( s' = (\bar{p}', z') \). Since private sector agents make intertemporal decisions, they must have beliefs about how policy is chosen in the future. Since the MA acts after observing \( s \), these beliefs are summarized as a money growth rule or policy rule, \( \chi(.): \sigma \rightarrow \mathcal{R}_+ \). This timing is illustrated in Figure 1.

2.1. Final Good Production

Final good firms are perfectly competitive and produce output using intermediate goods as input. Final goods are consumed by households. The representative firm’s problem is:

\[
\max_{\{y_i\}} P \left[ \int_0^1 y_i^{(\lambda-1)/\lambda} di \right]^{\lambda/(\lambda-1)} - \int_0^1 P_i y_i di, \quad \lambda > 1. 
\]

\(^3\)Discussion regarding the extension of results to other equilibrium concepts such sustainable equilibrium are contained in Section 8.
Here: $P$ is the price of final output; $P_i$ is the input price set by intermediate good firm $i \in [0, 1]$; and $\lambda$ is the elasticity of substitution across intermediate goods. The first-order necessary condition (FONC) for this problem states the familiar ‘demand as a function of relative price’ condition:

$$y_i = \left( \frac{P}{P_i} \right)^\lambda y, \quad y \equiv \left[ \int_0^1 y_i^{(\lambda-1)/\lambda} \, di \right]^{\lambda/(\lambda-1)}.$$

2.2. Intermediate Good Production

Intermediate good firms produce goods using labor according to $y_i = h_i$. Labor is hired at the competitive wage $W$. Linearity in production implies that the nominal wage is exactly the firm’s nominal marginal cost.

Price rigidity is introduced via the decision-making constraints of these firms. Suppose firms have only one option in their pricing decision: choose a single price to charge in the current and following period after observing the PS state, $(s, X)$. This is the standard, two-period Taylor (1980) form of price stickiness found in the monetary business cycle literature (see, for example, the textbook treatment of Romer, 2001, ch. 6), and is the specification used in King and Wolman (2004), hereafter KW.

In this paper, a firm making its pricing decision has two options: (a) be sticky, and choose a single price after observing $(s, X)$ for the current and following period; or (b) be flexible, and choose one price for the current period after observing $(s, X)$, and another price in the next period after observing $(s', X')$. Choosing option (b) requires paying a fixed cost.\footnote{\footnotesize This is equivalent to assuming that firms pay a fixed cost for every price change, but that they are forced to do so at least every second period.} This fixed cost corresponds to the incremental decision-making and implementation cost of one additional price change within the same duration of time. Firms choose their frequency of price change in response to expected inflation. This specification is chosen for the sake of expositional clarity. The key results are robust to and, in fact, strengthened in more elaborate specifications of endogenous price rigidity, such as those used in the state-dependent pricing literature; see Sections 7 and 8.
2.2.1. Sticky prices  If a firm chooses to set a sticky price, it chooses a single price, $\bar{P}$, to maximize two-period discounted profits:

$$\bar{\Upsilon} \equiv \max_{\bar{P}} \left[ \alpha \left( P^\lambda \bar{P}'^{1-\lambda} y - WP^\lambda \bar{P}'^{-\lambda} y \right) + \beta \alpha' \left( P'^\lambda \bar{P}'^{1-\lambda} y' - W'P'^\lambda \bar{P}'^{-\lambda} y' \right) \right],$$

where primes (') denote one-period-ahead variables. Here, the final good firm’s demand function has been substituted in, and $\alpha$ is the marginal value of current profit to the representative household. I denote the sticky price set today with a prime since this is the price (after normalization) that is inherited in the MA state, $s'$, in the following period. In a symmetric equilibrium, all sticky price firms charge the same two-period price:

$$\bar{P}' = \hat{\lambda} \left( \frac{\alpha P^\lambda y W + \beta \alpha' P'^\lambda y' W'}{\alpha P^\lambda y + \beta \alpha' P'^\lambda y'} \right), \quad \hat{\lambda} = \frac{\lambda}{\lambda - 1},$$

which is a markup, $\hat{\lambda}$, over the weighted sum of current and future marginal cost.

2.2.2. Flexible prices  If a firm chooses to be flexible, it chooses a price to charge today, $\tilde{P}$, and a price to charge tomorrow, $\tilde{P}'$, according to:

$$\tilde{P} = \hat{\lambda} W, \quad \tilde{P}' = \hat{\lambda} W'.$$

Since these prices are chosen after observing the MA’s action in each period, they are set optimally as a markup over observed marginal cost.

To set flexible prices, a firm must pay a fixed cost, $\phi$. This represents the units of labor it will expend to set a new price after observing $M'$. A firm will choose option (b) over option (a) if the difference in discounted two-period profits is greater than the fixed cost. That is, a firm with fixed cost $\phi_i$ will choose to be flexible if:

$$\alpha \tilde{\Pi} + \beta \alpha' \left[ \tilde{\Pi}' - W' \phi_i \right] \geq \bar{\Upsilon},$$

where $\tilde{\Pi} = \left( \tilde{P} - W \right) \left( P / \tilde{P} \right)^\lambda y$ denote optimized one-period profits. Call this condition the cut-off condition.

The CDF for the fixed cost among firms making their pricing decisions is denoted $F(\phi)$. A primary goal of this paper is to show that for plausible magnitudes of $\phi$, firms choose to
be flexible as opposed to sticky in high inflation equilibria. Hence, the exact specification of the distribution is not important; simply that the support is bounded with a maximal value, $\phi_{\text{max}}$. Denote the value of the fixed cost that satisfies the cut-off condition with equality as $\phi^*$. All firms with $\phi_i \leq \phi^*$ choose to set flexible prices, while all others set sticky prices. The fraction of firms currently making pricing decisions that choose flexibility is denoted as $z' = \mathcal{F}(\phi^*)$. I denote this with a prime since this is the fraction of current price-setters choosing flexibility that is inherited by the MA in the following period. If the cut-off condition holds with inequality at $\phi_{\text{max}}$, then $z' = 1$.

2.3. Households

Households value consumption $(c)$ and labor $(h)$ according to:

$$\sum_{t} \beta^t [\log (c) - \psi h], \quad 0 < \beta < 1, \ \psi > 0.$$  

The household faces two sequences of constraints. The first is the flow budget constraint:

$$M_t + B_t \leq R_{t-1}B_{t-1} + M_{t-1} - P_{t-1}c_{t-1} + (1 + \theta_{t-1}) \left( W_{t-1}h_{t-1} + \int_{0}^{1} \Pi_{i,t-1} di \right) + T_t.$$  

This is relevant during securities trading in each period $t$. Here: $B_t$ is nominal bond holdings that pay a gross return of $R_t$ upon maturity at date $t+1$; $M_t$ is the value of money holdings; $P_t$ is the consumption good price; $W_t$ is the nominal wage rate; $\Pi_{i,t}$ are nominal profits from firm $i \in [0,1]$; and $T_t$ is a lump-sum transfer from the MA. Finally, $\theta_t$ is a subsidy to production income.

After securities trading, households interact with firms in the goods and labor markets. The household supplies labor at the wage $W_t$, and buys consumption at the price $P_t$. Consumption purchases are subject to a cash-in-advance constraint:

$$M_t \geq P_t c_t, \ \forall t.$$  

4 Indeed, the distribution can be degenerate at $\phi_{\text{max}}$ without altering any of the key results. Allowing for a smooth CDF aids both in exposition (since it eliminates discrete jumps from full rigidity to full flexibility) and in numerical computation of equilibrium.

6 Trading occurs after observing the PS state, so this model displays the standard ‘Lucas timing’ of events within a period. See Lucas (1982) and the textbook treatment of Sargent (1987), ch. 5.
The household’s intertemporal FONC is:

\[ \frac{1}{P_c} = \beta R \frac{1}{P\tilde{c}}. \]

In equilibrium, \( R \geq 1 \), so that the cash-in-advance constraint holds with equality. Substituting this into the FONC delivers:

\[ \chi (s) = \frac{M'}{M} = \beta R. \]

That is, in equilibrium, the rate of nominal interest reflects the expected rate of money growth relative to time preference.

The household’s intratemporal FONC is:

\[ \frac{1}{c} \left( 1 + \frac{\theta}{R} \right) \frac{W}{P} = \psi. \]

Absent the subsidy (\( \theta = 0 \)), a non-zero nominal interest rate drives a wedge between the real wage and the marginal rate of substitution in consumption and labor. This interest rate distortion represents the fact that expected future inflation erodes the return to current labor effort in cash-in-advance models (see Cooley and Hansen, 1989).

I set \( \theta = R - 1 \) to eliminate this distortion. I do this for two reasons. First, the cost of expected future inflation cannot be influenced by the current MA; eliminating this makes clear that it is the welfare trade-off between the current benefit of unexpected inflation and the current cost of realized inflation that characterizes monetary discretion. Second, setting \( \theta = R - 1 \) and using the cash-in-advance constraint, the intratemporal FONC becomes:

\[ W = \psi M, \]

so that in equilibrium, the growth rate of the nominal wage between any two periods, \( t \) and \( t + 1 \), is determined by the money growth rate between \( t \) and \( t + 1 \). This ensures that across low and high inflation regimes, aggregate price level inflation is appropriately reflected in the growth rate of wages/marginal cost.\(^7\) Finally, note that with \( \theta = R - 1 \), the marginal value of current profit is given by \( \alpha = 1/P_c. \)

\(^7\) This discussion makes clear that it would be inappropriate to consider ‘real rigidities’ (see, for example, Ball and Romer, 1991) in the current analysis. Such considerations typically manifest themselves in the
2.4. Government Budget Constraint

The budget constraint faced by the MA is:

$$T_t = M_t - M_{t-1} - \theta_{t-1} \left( W_{t-1} h_{t-1} + \int_0^1 \Pi_{i,t-1} di \right), \; \forall t.$$  

The lump-sum transfer to the household finances the monetary injection, net of the subsidy to production income. The MA does not issue or purchase nominal bonds, so these are in zero net supply.

3. PRIVATE SECTOR EQUILIBRIUM

Though ultimate interest is in characterizing Markov perfect equilibrium (MPE), I first define a private sector equilibrium (PSE) in which the MA’s current action, $X$, and future policy, $\chi$, need not be welfare maximizing. In the definition, lower-case variables denote nominal variables determined in the current period normalized by the current money stock, e.g. $p \equiv P/M$, $\bar{p} \equiv \bar{P}/M$, $\tilde{p} \equiv \tilde{P}/M$, etc. I do this since all equilibria are neutral in the usual sense: if the initial money stock is doubled, a PSE exists in which all real allocations are identical and only nominal variables are doubled.

Given that firms and households make intertemporal decisions, these agents must have expectations about how policy is determined in the future. This is given by the policy rule, $\chi$. Moreover, anticipating results, certain policy rules admit multiple PSE. As a result, agents must understand how expectations are coordinated across equilibria. Since attention is restricted to perfect foresight environments, this amounts to understanding which equilibrium will prevail in each period. In much of the analysis to follow, the policy rule will admit two equilibria: an optimistic equilibrium with low inflation, and a pessimistic equilibrium with high inflation. I introduce an indicator, $\zeta$, that takes on one of two values, $\zeta \in \{lo, hi\}$, denoting whether expectations are coordinated on the optimistic or pessimistic household’s intratemporal FONC. In equilibrium, this would generate divergences between money growth and marginal cost growth that would not be appropriate for the study of perfectly anticipated, trend inflation due to discretion in monetary policy.
equilibrium occurring in the current and all future periods. In the following definition, I index private sector decision rules by both $\chi$ and $\zeta$ to emphasize this dependence of current behavior on expectations.

**Definition 1** Given beliefs (a policy rule) $\chi$ and (an indicator) $\zeta$, for all PS states $(s, X)$, a private sector equilibrium is a set of allocation rules \( \{c(s, X; \chi, \zeta), h(s, X; \chi, \zeta)\} \), pricing rules \( \{\bar{p} (s, X; \chi, \zeta), \tilde{p} (s, X; \chi, \zeta), z'(s, X; \chi, \zeta)\} \), and prices \( \{R(s, X; \chi, \zeta), p(s, X; \chi, \zeta)\} \) such that: households are optimizing, prices are set optimally, $z'(s, X; \chi, \zeta)$ satisfies the cut-off condition, the goods, labor, and bond markets clear, and $R(s, X; \chi, \zeta) \geq 1$.

By Walras’ Law, the money market clears.

In the rest of this section, I provide a more compact characterization of PSE. First, the household’s intratemporal FONC states that the normalized wage is constant, $w = \psi$. Hence, the normalized flexible price is also constant, $\tilde{p} = \hat{\lambda} \psi$.

Final good firm maximization generates the following normalized price level equation:

$$p(s, X; \chi, \zeta) = \left\{ \frac{1}{2} \left[ (1 - z) \left( \frac{\bar{p}}{X} \right)^{1-\lambda} + (1 - z') \tilde{p}'^{1-\lambda} + (z + z') \left( \hat{\lambda} \psi \right)^{1-\lambda} \right] \right\}^{\frac{1}{1-\lambda}},$$

where $z' = z'(s, X; \chi, \zeta)$ and $\tilde{p}' = \tilde{p}'(s, X; \chi, \zeta)$. From the cash-in-advance constraint:

$$c(s, X; \chi, \zeta) = 1/p(s, X; \chi, \zeta).$$

The labor market clearing condition is:

$$h(s, X; \chi, \zeta) = \frac{\tilde{p}^{\lambda - 1}}{2} \left[ (1 - z) \left( \frac{\bar{p}}{X} \right)^{-\lambda} + (1 - z') \tilde{p}'^{-\lambda} + (z + z') \left( \hat{\lambda} \psi \right)^{-\lambda} \right] + \frac{1}{2} \int_{0}^{\hat{F}^{-1}(\zeta)} \phi dF(\phi),$$

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8 It is also possible for expectations to fluctuate deterministically over time. In this case, $\zeta$ represents an entire sequence of time-indexed indicators. Stochastic equilibria can also be constructed, in which case $\zeta$ represents a sunspot shock. Given that the primary objective of this paper is to determine when equilibrium is unique, I focus on the simple case described above.

9 For simplicity, I do not index $\chi$ by $\zeta$ in Definition 1 since PSE is defined for arbitrary policy. That is, future money growth is assumed to depend only on $s'$, independent of which equilibrium is expected. In the analysis of MPE, $\chi$ will be indexed by $\zeta$, since the optimal policy response of the MA will depend on private sector behavior, which depends on coordination of expectations.
where \( p = p(s, X; \chi, \zeta) \). Finally, the intertemporal FONC bounds the set of feasible PSE money growth rules; \( R(s, X; \chi, \zeta) = \chi(s') / \beta \), so that \( \chi(s) \geq \beta \) for all \( s \).

Hence, equilibrium \( p(\cdot), c(\cdot), h(\cdot) \), and \( R(\cdot) \) are determined residually from \( \bar{p}'(\cdot) \) and \( z'(\cdot) \). These are determined as follows. The FONC for sticky price-setting implies that:

\[
\bar{p}' = \hat{\lambda} \psi \left( \frac{p^{\lambda-1} + \beta p'^{\lambda-1} \chi(s')^{\lambda}}{p^{\lambda-1} + \beta p'^{\lambda-1} \chi(s')^{\lambda-1}} \right), \tag{4}
\]

where \( p' = p(s', \chi(s'); \chi, \zeta) \). Finally, the cut-off condition states that \( z' = \mathcal{F}(\phi^*) \) satisfies:

\[
\frac{p^{\lambda-1}}{(\hat{\lambda} \psi)^{\lambda}} \left( \hat{\lambda} - 1 \right) \psi + \beta \left[ \frac{p'^{\lambda-1}}{(\hat{\lambda} \psi)^{\lambda}} \left( \hat{\lambda} - 1 \right) - \phi^* \right] \psi \geq \frac{p^{\lambda-1}}{p^{\lambda} \chi(s')^{\lambda}} (\bar{p}' - \psi) + \beta p'^{\lambda-1} \left( \frac{\chi(s')^{\lambda}}{p'} \right) \left( \frac{p'}{\chi(s')} - \psi \right). \tag{5}
\]

This holds with strict equality whenever \( \phi^* < \phi_{\text{max}} \), and with weak inequality whenever \( \phi^* = \phi_{\text{max}} \). Conditions (4) and (5) characterize PSE \( \bar{p}' \) and \( z' \). Remaining PSE objects are determined as described above. This simplifying result is summarized as follows:

**Proposition 2** Given beliefs \( \chi \) and \( \zeta \), a PSE is characterized as decision rules, \( \bar{p}' \equiv P(s, X; \chi, \zeta) \) and \( z' \equiv Z(s, X; \chi, \zeta) \), such that for all \( (s, X) \), equations (4) and (5) are satisfied.

From the equilibrium conditions it is clear that the only pay-off relevant variables inherited by the current MA from the past are the previous period’s pricing decisions. Hence, when attention is turned to MPE, the MA state is \( s = (\bar{p}, z) \).

Moreover, when the MA inherits fully flexible prices, current money growth is neutral. The intuition for this is obvious.\(^{10}\) With no inherited price stickiness, current money growth has no effect on the monopoly distortion or the relative price distortion, since this influence requires the presence of sticky prices.

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\(^{10}\)To see this in the equilibrium characterization, note that when \( z = 1 \), current money growth, \( X \), has no direct influence on the normalized price level, \( p \) (see equation (1)). But since \( X \) enters the decision rules, \( P \) and \( Z \), only indirectly via \( p \) (see equations (4) and (5)), there is no influence of current money growth on \( \bar{p}' \) and \( z' \) when \( z = 1 \). Hence, there is no effect on consumption or hours worked (see equations (2) and (3)).
4. MULTIPlicity OF EQUiLIBRIUM

Having characterized PSE, it is possible to illustrate the potential for multiple equilibria. This section demonstrates that multiplicity of PSE stems from a strategic complementarity across firms’ pricing decisions for particular monetary policy rules. Monetary policy will satisfy these conditions whenever it is sufficiently accommodative of private sector expectations. It is this multiplicity of PSE that translates into multiple MPE when the MA is maximizing. This is taken up in Sections 5 and 6.

4.1. Strategic Complementarity in Price-Setting

This strategic complementarity is first illustrated by KW and I discuss it here for completeness. To do so, it is easiest to work with their model, which is a special case of that presented here. In KW, choosing to increase the frequency of price change (i.e., charging one price for each period, as opposed to charging one price for two periods) is infinitely costly. All firms act as sticky price firms in PSE. The MA and PS states are reduced to \( s = \bar{p} \) and \( (s, X) = (\bar{p}, X) \), respectively.

Consider the following rewriting of the FONC for sticky price-setting:

\[
\bar{p}' = \lambda \left[ (1 - \gamma) \psi + \gamma \chi (\bar{p}') \psi \right].
\]

The normalized two-period price is a markup over a weighted average of current and future marginal cost, where the relative weight on future marginal cost is given by:

\[
\gamma = \frac{\beta p' \lambda^{-1} \chi (\bar{p}')^{\lambda-1}}{p \lambda^{-1} + \beta p' \lambda^{-1} \chi (\bar{p}')^{\lambda-1}} \in (0, 1).
\]

In the expression for \( \gamma \), the current price level is given by (1) with \( z = z' = 0 \), and the future price level is:

\[
p' = \left\{ \frac{1}{2} \left[ \left( \frac{\bar{p}}{\chi (p')} \right)^{1-\lambda} + \bar{p}'^{1-\lambda} \right] \right\}^{\frac{1}{1-\lambda}}.
\]

\[11\]Note that this differs from the multiplicity result of Albanesi et al. (2003), where given policy, PSE is unique. Instead, their framework generates multiple solutions to the MA’s problem that are rationalized by private sector expectations. See their paper and King and Wolman (2004) for discussion.
where the one-period ahead sticky price, $\bar{p}_{0}''$, is given by the decision rule (4) with $z = z' = 0$. Hence, the weight can be viewed as a function of the sticky price, the PS state, and beliefs about future money growth: $\gamma = \gamma (\bar{p}; \bar{p}, X; \chi)$. For now I ignore the issue of how expectations are coordinated in the current period; the point is to determine when this matters by viewing equation (6) as a function of $\bar{p}'$ with potentially multiple solutions. However, the coordination of expectations in the future matters for $\bar{p}_{0}''$, via the effect of $\bar{p}_{0}''$ on $\gamma$. This relevance of future expectations on current period behavior is discussed shortly.

Following KW, I interpret equation (6) as the best response function for an individual firm: given $(\bar{p}, X)$ and $\chi$, this maps out the optimal price for firm $i$, $\bar{p}_{i}'$, as a function of the price of all other price-setting firms, $\bar{p}_{j}'$, for all $j \in [0, 1], j \neq i$. Specifically:

$$\bar{p}_{i}' \equiv f (\bar{p}_{j}' ; \bar{p}, X; \chi) = \lambda \left\{ [1 - \gamma (\bar{p}_{j}' ; \bar{p}, X; \chi)] \psi + \gamma (\bar{p}_{j}' ; \bar{p}, X; \chi) \chi (\bar{p}_{j}') \psi \right\}. \quad (7)$$

Firm $i$’s optimal price depends on the price set by all other firms $j$ through two channels: (1) future marginal cost, via the effect of $\bar{p}_{j}'$ on future money growth, $\chi$; and (2) the relative weight placed on future marginal cost, via the effect of $\bar{p}_{j}'$ on $\gamma$. PSE requires $\bar{p}_{i}' = \bar{p}_{j}'$.

If $\partial f / \partial \bar{p}_{j}' > 0$, then there exists strategic complementarity: the higher is the price set by other firms, the higher is the optimal price for any individual firm (see Cooper and John, 1988). If this complementarity is sufficiently strong, there may be multiple crossings of equation (7) with the 45°-line, and hence, multiple equilibria. Whether this is the case depends wholly on the policy rule, $\chi$.

As a benchmark, consider $\chi (\bar{p}) \equiv 1$, the case in which the MA always delivers zero money growth. This corresponds to the first-best policy achieved under commitment (see King and Wolman, 1999). Since future money growth does not respond to current price-setting, marginal cost is constant across periods ($W''/W = \chi (\bar{p}) = 1$). Regardless of the price set by other firms, an individual firm’s best response is simply the static markup rule,

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12 The intuition is straightforward. ‘Money demand’ distortions associated with inflation greater than the Friedman Rule are eliminated by the subsidy, $\theta$. With commitment, the MA has no influence on the monopoly distortion since inflation is perfectly anticipated. Hence, the only distortion affected by policy is the relative price distortion; this is eliminated with zero inflation (zero money growth).
\( \bar{p}_i \equiv f (\bar{p}_j; \bar{p}, X; \chi) = \hat{\lambda} \psi. \) With zero money growth there is no complementarity. This is displayed in Figure 2. In this case, how expectations are coordinated is not relevant.

As a second example, consider the case when money growth is an increasing, linear function of prices, \( \chi (\bar{p}) = a_0 + a_1 \bar{p}, \ a_1 > 0. \) As firms set higher prices, \( \bar{p}_j \), future marginal cost, \( \chi (\bar{p}_j) \psi, \) rises. Moreover, as \( \chi \) rises, \( \gamma \rightarrow 1, \) so that the relative weight on future marginal cost rises too. The optimal price for a firm is increasing in the price set by other firms. Complementarity exists because the policy rule responds positively to – or accommodates – the pricing decision of firms.

In Figure 3, I illustrate this for \( \chi (\bar{p}) = 0.302 \times \bar{p}. \) Consider either the solid line or the dashed line. The best response function first crosses the 45º-line from above. As \( \bar{p}_j \) increases so too does the slope; the slope eventually exceeds one so there is a second crossing of the 45º-line from below. Because \( \gamma \rightarrow 1 \) as \( \bar{p}_j \) rises, and because \( \partial \chi / \partial \bar{p} = a_1: \)

\[ \lim_{\bar{p}_j \to \infty} \partial f / \partial \bar{p}_j = \hat{\lambda} \psi a_1. \]

Hence, when the MA’s policy rule is linear, a necessary condition for multiplicity is that \( \hat{\lambda} \psi a_1 > 1. \) Moreover, when multiple crossings exist, there are exactly two of them: an optimistic equilibrium with current expectations coordinated on low inflation (and actions coordinated on low price-setting), and a pessimistic equilibrium with current expectations coordinated on high inflation.

Also, the best response function depends on how expectations are coordinated in the future. This is because the relative weight, \( \gamma, \) depends on the sticky price set next period, \( \bar{p}''; \bar{p}'' \) in turn depends on \( \bar{p}'', \) and so on. Hence, private sector agents must understand which equilibrium will prevail today and in all subsequent periods.

This is illustrated in Figure 3. The solid line displays the best response function when future sticky prices are determined by the decision rule, \( P, \) with \( \zeta = \text{lo}. \) If agents expect low inflation today, this results in the crossing marked with the diamond. Given that \( \zeta \) defines

\footnote{Here are some numerical details with additional discussion contained in Appendix A. I set \( \beta = 0.98, \ \lambda = 11, \) and \( \psi \) so that \( h_{ss} = 0.3 \) in the zero-inflation steady state.}

\footnote{This is not sufficient since it is possible that (7) lies everywhere above the 45º-line, so that no PSE exist.}
constant expectations coordination over time, this is a PSE since it applies the same decision rule used in the future to the current period; that is, at this crossing, \( \bar{p}' = f (\bar{p}', \bar{p}, X; \chi) = P (\bar{p}, X; \chi, \text{lo}) \).\(^{15}\) The dashed line displays the best response function when future prices are determined by \( P \) with \( \zeta = \text{hi} \); for these beliefs, the PSE is marked with the square.

Finally, Figure 4 illustrates that the exact number of crossings depends on the shape of the policy rule \( \chi \). Here, \( \partial^2 \chi / \partial \bar{p}^2 \) is initially positive, but beyond an inflection point is negative. As a result, the number of crossings is three. Figure 4 displays the best response function when future expectations are coordinated on the lowest inflation equilibrium.

### 4.2. Discussion

Before proceeding, I provide a few comments on the source of complementarity. First, when attention is turned to monetary discretion, the policy rule will be an increasing function of the normalized sticky price. That is, a benevolent MA will find it optimal to accommodate the private sector’s expectations and pricing decisions.

Second, it is insightful to contrast the strategic complementarity highlighted here and that discussed in Ball and Romer (1991). There are three key differences. First, in Ball and Romer the complementarity arises due to *endogenous price rigidity*, in particular, the decision of firms to alter prices in a state-contingent manner. Here, the complementarity operates through the *exogenously rigid* two-period Taylor price. Second, Ball and Romer’s model is static, and the complementarity operates through a feedback of current price-setting, through *current marginal cost*, into the pricing decision of individual firms. Here, current (normalized) marginal cost is pinned down as \( \psi \), but *future marginal cost*, \( \chi (\bar{p}') \psi \), responds via the increasing function, \( \chi \). The last and most important difference is that in Ball and Romer, the feedback on pricing decisions is due to *real rigidity* in marginal cost. Here, firms’ expectations about other firms’ actions, and – crucially – the *accommodative monetary policy* response to those actions, feeds back into pricing decisions.

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\(^{15}\)The second crossing of the solid line with the 45°-line represents a PSE with expectations changing over time. Here, agents understand that the high inflation equilibrium prevails today, but the low inflation equilibrium will prevail for all future periods.
Finally, an important element to the complementarity is the effect of price-setting on the relative weight placed on future marginal cost. As firms raise prices, future marginal cost rises and an individual firm cares more about the future in its price-setting. This is because the firm’s profit function is asymmetric across having a relative price that is too high versus one that is too low. This asymmetry is discussed in detail in Devereux and Siu (2005), and can be understood through the following extreme, but intuitive thought experiment.

Suppose future money growth, $\chi\left(\bar{p}_j^t\right)$, is high. Further, suppose firm $i$ must decide between pricing as a markup over current marginal cost, $\bar{p}_i^t = \hat{\lambda}\psi$, or future marginal cost, $\bar{p}_i^t = \hat{\lambda}\chi\left(\bar{p}_j^t\right)\psi$. In either case, it earns static optimal profits in one of the two periods.

By choosing the latter price, the firm earns optimal profits in the second period, but its current price is high relative to firms that set their price in the previous period. As a result, the firm’s first period demand and profit will be low, but bounded above zero. Now suppose the firm prices to current marginal cost; it earns optimal profits in the first period, but its future relative price will be low. This implies that the firm’s second period demand will be high, in the same period when its profit margin is negative (at least for sufficiently high marginal cost growth).

Hence, the firm prefers to set a price that is too high relative to one that is too low; the firm sacrifices current profit to ensure non-negative profits in the future. It is this relative price effect on a firm’s demand that makes the weight, $\gamma$, increasing in $\bar{p}$.\(^{16}\) As long as the policy rule, $\chi$, is increasing in $\bar{p}$, the asymmetry in profits acts as a force for strategic complementarity in price-setting behavior.

### 4.3. The Case with Endogenous Price Rigidity

With exogenous price rigidity the only action firms can take to guard against high future inflation is to set a high price in the present. However, it is plausible to think that there are other defensive actions firms take when faced with high inflation. One is to reset prices.

\(^{16}\)Of course, this example is extreme since any firm, given the opportunity, would shut down rather than meet demand at a negative profit margin. However, the intuition holds for any positive value of money growth. Again, see Devereux and Siu (2005) for discussion.
more often so that at any point in time, prices are more ‘in line’ with aggregate conditions. Here, I consider a simple example to illustrate that the fraction of firms choosing flexibility increases with expected future inflation.\footnote{See also Devereux (1987) for a model which considers the effect of monetary variability on the endogenous degree of wage indexation, and its impact on discretionary monetary policy.}

Let the MA’s policy rule be \( \chi(\bar{p}, z) = 0.302 \times \bar{p} \), and let the fixed cost of price change be uniformly distributed, \( F(\phi) = U[0, \phi_{\text{max}}] \). For different values of the two-period price, \( \bar{p}_j^t \), I use the cut-off condition (5) to determine the fraction of firms that prefer flexibility, \( z' \), as opposed to charging \( \bar{p}_j^t \). Then the FONC (4) is used to determine the best response sticky price, \( \bar{p}_i^t \). Figure 5 plots \( \bar{p}_i^t \) and \( z' \) for two values of \( \phi_{\text{max}} \): the first column sets the maximal fixed cost to 20\% of per-period firm revenue in the zero-inflation steady state, and the second column sets this to 10\%. For simplicity, I plot only the case when agents expect the low inflation equilibrium in future periods. Remaining parameter values are as specified in Subsection 4.1, and details on computation are in Appendix A.

Allowing for endogenous price rigidity does not qualitatively change the best response function. There are two crossings of the 45°-line. The first is optimistic, featuring expectations of low inflation and a small degree of price flexibility; the second is pessimistic, with expectations of high inflation and greater flexibility. As \( \phi_{\text{max}} \) falls, an increasing number of firms choose to be flexible for given future inflation, as shown in the second column. For sufficiently high values of inflation, all firms choose flexibility, \( z' = 1 \). At this point the best response function ceases to be ‘relevant’ since no firms actually set sticky prices.\footnote{This makes interpretation of the policy rule, \( \chi(\bar{p}) \), in this example somewhat difficult. This is no longer a problem when I consider a maximizing policy authority whose rule, \( \chi(\bar{p}, z) \), is a function of both sticky prices and the fraction of firms charging that price. See Section 6.}

\section{5. A MAXIMIZING MONETARY AUTHORITY}

The MA’s objective is to maximize the present discounted value of household utility from the current period forward through the choice of current money growth, \( X \). The MA takes past decisions and its future incarnation’s policy rule, \( \chi \), as given and beyond its
control. This is the expression of the time-consistency problem as articulated by Kydland and Prescott (1977): the current MA is unable to compel its future self to appropriately account for the effect of its policy on current private sector expectations and decisions.

Here, the time-consistency problem takes on an added dimension. Current private sector behavior depends on expectations of future policy, given by $\chi$. But certain policy rules admit multiple PSE. This implies that the current MA must also take the coordination of expectations as given. In its policy problem, the MA is ‘trapped’ by inflation expectations.

The MA’s problem can be stated as follows:

$$\max_{\chi} \left[ U(s, X; \chi, \zeta) + \beta U(s', \chi(s'); \chi, \zeta) + \beta^2 U(s'', \chi(s''); \chi, \zeta) + ... \right], \quad \forall s \in \sigma,$$

taking as given $\chi$ and $\zeta$. Here: $s = (\bar{p}, z)$, $U(s, X; \chi, \zeta) = \log c(s, X; \chi, \zeta) - \psi h(s, X; \chi, \zeta)$, $s' = (P(s, X; \chi, \zeta), Z(s, X; \chi, \zeta))$, and so on; $P(.)$ and $Z(.)$ are defined by equations (4) and (5); and $c(.)$ and $h(.)$ are defined by equations (2) and (3). A MPE can be defined as a PSE and a policy rule, $\chi : \sigma \to \mathbb{R}_+$, that solves the MA’s problem. The MPE policy rule, $\chi(s; \zeta)$, differs across values of $\zeta$ since the MA takes expectation coordination as given.

Here, I consider an alternative definition of MPE and derive the generalized Euler equation (GEE), both of which are due to Klein et al. (2004).$^{19}$

Definition 3 Given $\zeta$, a Markov perfect equilibrium consists of a value function, $V$; decision rules, $P$ and $Z$; and a policy rule, $\chi$, such that for all $s = (\bar{p}, z) \in \sigma$:

- given $\chi(s; \zeta)$, $P(s, X; \chi, \zeta)$ and $Z(s, X; \chi, \zeta)$ are the PSE decision rules characterized in Proposition 2;

- given $P(s, X; \chi, \zeta)$, $Z(s, X; \chi, \zeta)$, and $V(s; \zeta)$:

$$\chi(s; \zeta) \in \arg\max_{\chi} [U(s, X; \chi, \zeta) + \beta V(P(s, X; \chi, \zeta), Z(s, X; \chi, \zeta); \zeta)];$$

- given $P(s, X; \chi, \zeta)$, $Z(s, X; \chi, \zeta)$, and $\chi(s; \zeta)$:

$$V(s; \zeta) = U(s, \chi(s; \zeta); \chi, \zeta) + \beta V(P(s, \chi(s); \chi, \zeta), Z(s, \chi(s); \chi, \zeta); \zeta).$$

$^{19}$For a related definition of time consistent equilibrium, see the appendix of Kydland and Prescott (1977). I thank Victor Rios-Rull for bringing this to my attention.
To conserve on notation, I do not index \( V \) by \( \chi \), since this is obvious; the value function is constructed using private sector decision rules which: (i) depend on (are indexed by) \( \chi \), and (ii) take the current period money growth rate as \( \chi(s, \zeta) \). This last restriction is correct, since the value function is used by the current MA in evaluating future welfare, taking future policy as given by \( \chi \). This definition concisely captures the notion of time consistency: the policy rule attributed to the choice of money growth by the future MA coincides with the optimizing choice of current money growth for all \( s \in \sigma \), given \( \zeta \).

The GEE is the FONC associated with MA optimization. This is useful both for interpretation and computation (see Appendix B). Wherever the policy and decision rules are differentiable, \( X \) must solve:

\[
U_c C_X + U_h H_X + \beta (V''_{\bar{p}} P_X + V''_z Z_X) = 0, 
\]

where

\[
C_X = c_X + c_{\bar{p}} P_X + c_z Z_X, \\
H_X = h_X + h_{\bar{p}} P_X + h_z Z_X, 
\]

for all \( s \in \sigma \), given \( \zeta \). Here, \( V'_{\bar{p}} \) represents the derivative of the one-period ahead value function with respect to its first argument, and \( V'_z \) with respect to the second. Likewise, \( c_i \) represents the derivative of the consumption allocation rule with respect to \( i \), for \( i = \{X, \bar{p}', z'\} \), and similarly for \( h_i \).

To express the GEE in terms of primitives, note from the definition of the value function:

\[
V'_{\bar{p}} = U'_c C_{\bar{p}} + U'_h H_{\bar{p}} + \beta (V''_{\bar{p}} P'_{\bar{p}} + V''_z Z'_{\bar{p}}). 
\]

To simplify this expression, rearrange equation (8) to obtain:

\[
\beta (V''_{\bar{p}} P'_{\bar{p}} + V''_z Z'_{\bar{p}}) = -X'_{\bar{p}} (U'_c C_X + U'_h H_X), 
\]

where \( X' = \chi(\bar{p}', z') \). As a result:

\[
V'_{\bar{p}} = U'_c C_{\bar{p}} + U'_h H_{\bar{p}} - \frac{P'_{\bar{p}}}{P'_X} (U'_c C_X + U'_h H_X). 
\]
Manipulating $V'_{z}$ in the same way and substituting, the GEE becomes:

$$U_{c}C_{X} + U_{h}H_{X} + \beta P_{X} \left[ U'_{c}C'_{p} + U'_{h}H'_{p} - \frac{P'}{P_{X}} (U'_{c}C_{X} + U'_{h}H_{X}) \right] + \beta Z_{X} \left[ U'_{c}C'_{z} + U'_{h}H'_{z} - \frac{Z'}{Z_{X}} (U'_{c}C_{X} + U'_{h}H_{X}) \right] = 0. \tag{9}$$

From a variational perspective, the MA’s policy choice affects current and one-period ahead utility; these marginal effects sum to zero at an optimum. In the current period, money growth affects the utility value of consumption via $U_{c}C_{X}$. This involves both direct and indirect effects. The direct effect, $c_{X}$, captures the erosion of the inherited normalized price, $\bar{p}$. This affects both the monopoly pricing distortion, and, for given $\bar{p}'$ and $z'$, the relative price distortion across firms. The terms $c_{p}'P_{X}$ and $c_{z}'Z_{X}$ capture the indirect effects on current pricing decisions, and hence, on the relative price distortion. Similarly, $U_{h}H_{X}$ captures the direct and indirect effects of current money growth on the utility value of labor.

The remaining terms in equation (9) represent the effect of current policy on the future via the future state, $s' = (\bar{p}', z')$. The first term involving square brackets, $\beta P_{X} [\cdots]$, can be split into two effects. The first is the indirect effect of the change in the future sticky price, $P_{X}$, on future utility values of consumption and labor, $U'_{c}C'_{p}$ and $U'_{h}H'_{p}$.

The second is the induced effect of a change in current policy on future policy. The term:

$$P_{X} \frac{P'}{P_{X}} = \frac{\partial \bar{p}'}{\partial X} \frac{\partial X'}{\partial \bar{p}'} = \frac{\partial X'}{\partial X}.$$

Hence, this reflects the change in future money growth due to current money growth’s influence on the sticky price, $\bar{p}'$. Similarly, the second square bracket term reflects the indirect and induced policy effects of $X$ on the future via the fraction of flexible prices, $z'$.

Note that the GEE must be satisfied wherever policy and decision rules are differentiable. However, there is no reason to restrict attention to policy rules that are continuous when $z = 1$. With full price flexibility, the current MA is indifferent between all values of $X$ since money is neutral. In the following section, I discuss how outcomes can differ when a discontinuity in the policy rule is allowed at $z = 1$. In Section 7, I consider an arbitrarily

20 These, in turn, involve direct and indirect effects via $C'_{p}$ and $H'_{p}$, analogous to those described above.
small perturbation of the model that implies that the optimal policy rule is necessarily discontinuous at $z = 1$. This results in unique MPE.

6. ANALYZING MARKOV PERFECT EQUILIBRIUM

Here I study MPE in calibrated versions of the model of Section 2. In Subsection 6.1, attention is restricted to differentiable policy rules. The first objective is to illustrate that two MPE exist. The second is to characterize the degree of price rigidity in the high inflation equilibrium. Subsection 6.2 considers the case with policy rules discontinuous at $z = 1$.21

The model calibration is standard. The period length is taken to be six months, so that no firm is forced to charge a price older than six months, even in the face of high inflation. I set $\beta = 0.98$ to accord with an annual risk-free real return of 4%. The demand elasticity of substitution, $\lambda$, determines the strength of the strategic complementarity. As in much of the sticky price literature, I choose $\lambda = 11$ as a benchmark value. This implies a price-to-marginal-cost markup of 10% in the zero inflation steady state (see, for e.g., Chari et al., 2000; Khan et al., 2003; KW; and Devereux and Siu, 2005). I also consider smaller values of $\lambda$ (higher markups) to capture the range of values used in the literature.22 The fraction of time spent in market activity in the zero inflation steady state is $h_{ss} = 0.3$.

6.1. Analysis of the Differentiable Case

I first consider the play of differentiable policy rules in which:

$$\chi (\tilde{p}, 1; \zeta) = \lim_{z \to 1} \chi (\tilde{p}, z; \zeta), \quad \forall \tilde{p},$$

despite the fact that the MA is indifferent between all values of $X$ at full price flexibility. To make this operational, I solve for MPE by approximating the MA’s policy rule by a

21 Again, I consider cases where expectations are coordinated on either the optimistic or pessimistic equilibrium occurring in all periods. Equilibria in which expectations fluctuate across low and high inflation can be constructed (see footnote 8). Since the emphasis of this paper is to show that the pessimistic equilibrium is fragile, I do not analyze this possibility.

22 For instance, $\lambda = 4.33$ in Dotsey et al. (1999), and $\lambda = 3.22$ in Christiano et al. (2005).
tensor product of Chebychev polynomials, which is differentiable by construction. I outline the algorithm developed to solve for MPE in Appendix B.

6.1.1. Exogenous price rigidity  When the fixed cost distribution is degenerate at infinity, no firm chooses to reset its price more frequently than once a year. Results from this version can be summarized as follows:

**Proposition 4** With exogenous price rigidity, the MPE policy rule, \( \chi(\bar{p}) \), is proportional in \( \bar{p} \). Hence, two locally isolated MPE exist: an optimistic equilibrium with low expected and realized inflation, and a pessimistic equilibrium with high inflation.

Discussion of this result is contained in KW, in their characterization of a homogenous money stock rule. The intuition is straightforward. From the PSE decision rules, (2) – (3), the direct effect of money growth on real outcomes is in direct proportion to the level of normalized preset prices, \( \bar{p} \). Moreover, \( \bar{p}' \) or \( z' \) depend on money growth only via its effect on the normalized price level, \( \bar{p} \), where again, the effect is proportional to \( \bar{p} \). As a result, optimal money growth, \( \chi(\bar{p}) \), is proportional to \( \bar{p} \).

Given this linearity, Subsection 4.1 shows that the number of PSE is exactly two. As a result, there are exactly two MPE, so that \( \zeta \in \{\text{lo}, \text{hi}\} \). In the steady state of the optimistic MPE, the inflation rate is 1.9\% per period (3.8\% per year), while real output is 0.04\% lower than in the zero inflation (first best) steady state. In the pessimistic MPE steady state, inflation is much higher at 13.8\% per period, and output is 1.91\% lower than with zero inflation. Hence, the pessimistic equilibrium can be interpreted as stagflation relative to the optimistic equilibrium.

6.1.2. Endogenous price rigidity  Here I consider a distribution of the fixed cost with bounded support, \([0, \phi_{\text{max}}]\). For the sake of computation, I choose \( \mathcal{F} \) to be uniform, though the exact specification is irrelevant to the results (see below).

From the GEE (9), the MA’s policy choice generates dynamic effects through its influence on the future MA state, \( s' = (\bar{p}', z') \). These effects (in particular, the fact that \( Z_X \neq \...
0) makes an analytical characterization of the optimal policy rule, \( \chi(\bar{p}, z; \zeta) \), infeasible. Instead, I characterize \( \chi \) numerically using the iterative algorithm of Appendix B. For a given \( \zeta \), the solution method converges to a unique MPE policy rule. This policy rule is a non-linear function of the fraction of flexible price firms, \( z \), but is proportional in the sticky price, \( \bar{p} \).

Again, the intuition is straightforward, since money growth appears in the PSE decision rules in direct proportion to \( \bar{p} \). Hence, two MPE exist, indexed by expectations.

The fraction of flexible price firms obviously increases with the inflation rate. The goal is to determine whether – for quantitatively reasonable values of \( \phi_{\text{max}} \) – prices are fully flexible in the steady state of the pessimistic, high inflation MPE.

To this end, I compute the pessimistic MPE for various values of \( \phi_{\text{max}} \) and find the largest value such that the steady state displays full price flexibility. That is, I find the value – call it \( \hat{\phi}_{\text{max}} \) – such that for all fixed cost distributions, \( F \), with \( \phi_{\text{max}} \leq \hat{\phi}_{\text{max}}, \ z = 1 \) in the steady state of the pessimistic MPE; for all \( F \) with \( \phi_{\text{max}} > \hat{\phi}_{\text{max}}, \ z < 1 \). Hence, the shape of \( F \) is irrelevant for finding \( \hat{\phi}_{\text{max}} \); if all firms choose to incur the fixed cost, \( \phi_i \in [0, \phi_{\text{max}}] \), for \( \phi_{\text{max}} \leq \hat{\phi}_{\text{max}} \), the exact distribution of those costs across firms does not matter.

Figure 6 plots the value of \( \hat{\phi}_{\text{max}} \) for various values of \( \lambda \). For the baseline calibration of \( \lambda = 11 \), \( \hat{\phi}_{\text{max}} = 8.9\% \) of semi-annual steady state firm revenue; i.e., as long as the cost of a single price change is less than 8.9\% of semi-annual revenue, all firms choose to incur it and the steady state of the high inflation equilibrium exhibits full price flexibility. As \( \lambda \) falls, so that the steady state markup increases, the cut-off value increases. For instance, when

\[ \text{Note that this is inherently a quantitative issue. That is, it cannot be that, for any finite value of } \phi_{\text{max}}, \text{a pessimistic MPE exists with full flexibility. To see this, compare the difference in gross profits from being flexible relative to being sticky, versus the value of } \phi_{\text{max}}. \text{Flexible price profits are simply the discounted two-period sum of static monopoly profits. For any future money growth rate, there is a finite lower bound on sticky price profits: a firm can always set its two-period price as an optimal markup over future marginal cost, and earn static monopoly profits in the second period of its price contract. The worst that can happen in the first period is that the firm’s relative price is so high that it generates zero demand and earns zero profit. Hence, the difference between flexible and sticky price profits is bounded. So as long as the maximal fixed cost is greater than this bounded difference, full price flexibility cannot be an equilibrium.} \]
the markup is calibrated to 25% ($\lambda = 5$), the cut-off value is $\hat{\phi}_{\text{max}} = 16.4\%$ of semi-annual revenue, and when the markup is 35% ($\lambda = 3.85$), $\hat{\phi}_{\text{max}} = 18.9\%$.

To understand this, note that as $\lambda$ decreases, so too does the strength of the strategic complementarity: intermediate goods become less substitutable, so a firm’s optimal price becomes less sensitive to others’ prices. For a pessimistic MPE to exist, it must exist at higher levels of money growth and inflation. At higher inflation, the greater is the benefit to choosing flexibility, and the greater is the degree of flexibility for a given fixed cost distribution. So as $\lambda$ decreases, the range of $\phi_{\text{max}}$ values for which pessimistic MPE displays full price flexibility increases.

Recall that the fixed cost, $\phi$, corresponds to the firm’s incremental cost of one additional price change. As such, the magnitude of $\hat{\phi}_{\text{max}}$ values in Figure 6 is large. It is much larger than those used in monetary business cycle models with state-dependent pricing. For instance, Dotsey et al. (1999) consider a value of $\phi_{\text{max}}$ equivalent to 1.5% of semi-annual steady state firm revenue, while Devereux and Siu (2005) consider a value of 2.85%.

More importantly, the magnitude of $\hat{\phi}_{\text{max}}$ is much larger than direct measures of the cost of a single price change. Zbaracki et al. (2004) is the leading study. They track the price-setting process of a multi-product manufacturing firm and quantify all fixed costs associated with the issuance of the firm’s price list – managerial (information-processing, decision-making), customer (communication, renegotiation), and physical ‘menu’ costs. At a semi-annual frequency, this comes to 2.5% of the firm’s revenue. It is clearly difficult to extrapolate based upon this single observation. For instance, it could be argued that Zbaracki et al.’s measure generates downward bias for inference of $\phi_{\text{max}}$, due to selection: by necessity, they study a firm that is willing to make annual price revisions in a low inflation environment. It is also easy to argue for upward bias. During periods of high inflation, it is likely that many of the tasks documented by Zbaracki et al. (market research, sales trips made expressly to communicate new prices, printing of price lists) would be made routine, less costly, or all-together eliminated. Hence, the relevant incremental cost of a price change, as it pertains to high inflation expectation traps, may be much smaller.

Nonetheless, the model’s results for the size of $\hat{\phi}_{\text{max}}$ are multiple times greater than
the calibrated and measured values discussed above. Hence, it seems likely that for any reasonable magnitude of $\phi_{\text{max}}$, prices are fully flexible in the high inflation equilibrium.

Finally, real output is actually higher in the pessimistic MPE than in the optimistic one. In the optimistic steady state, prices are less than fully flexible, and output is lower than in the zero inflation first-best. This is because in MPE, all inflation is perfectly forecasted. As a result, the output gains due to inflation’s effect on the monopoly distortion are outweighed by the losses due to the relative price distortion. But because prices are fully flexible in the pessimistic case, output is identical to that of the zero inflation steady state.\(^\text{25}\) Hence, the predictions for real outcomes are opposite to those from the model with exogenous price rigidity. This belies the interpretation of pessimism as periods of stagflation.

### 6.2. Allowing for a Discontinuity in the Policy Rule

When the MA inherits no sticky prices, current money growth has no real effect and the MA is indifferent between all values of $X$. This strict indifference opens up the possibility for a rich set of pessimistic MPE.

Since the principal objective is to demonstrate the fragility of self-fulfilling high inflation equilibria, I provide brief description here, with more detailed analysis in Appendix C. Consider the following discontinuous policy rule:

$$\hat{\chi} (\bar{p}, z; \zeta) = \begin{cases} 
\chi (\bar{p}, z; \zeta) & \text{for all } \bar{p} \text{ and } z < 1 \\
\hat{X} & \text{for all } \bar{p} \text{ and } z = 1 
\end{cases}$$  \hspace{1cm} (10)

where $\chi (\bar{p}, z; \zeta)$ is the differentiable MPE policy rule of Subsection 6.1 and $\hat{X} \geq \beta$. Obviously this rule is optimal for the MA: for all $z < 1$, this rule coincides with the original MPE rule; at $z = 1$, any value of $\hat{X}$ is optimal by indifference.

To ensure this is a MPE policy rule, all that needs to be checked is that private sector best responses constitute equilibrium behavior. For all $z < 1$, rule (10) coincides with the

\(^{25}\)Moreover, hours worked is greater in the pessimistic case relative to zero inflation since, in order to achieve full flexibility, labor input must be devoted to price change. Finally, note that when flexibility is less than full, the specification of $\mathcal{F}$ will matter for real variables via the equilibrium value of $z$.  

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original MPE rule, so the decision rules, $P$ and $Z$ given by (4) and (5), are optimal by definition. At $z = 1$, pricing decisions are independent of current money growth. To ensure equilibrium, all that needs to be checked is that all firms that chose flexibility in the previous period under the original policy rule continue to do so under rule (10). That is, it must be that at $z = 1$, no firm that chose flexibility finds it profitable to deviate to stickiness.

Since firms are differentiated only by their fixed cost, it suffices to check this for the firm with the highest fixed cost, $\phi_i = \phi_{\text{max}}$. The profitability of such a deviation depends on the value of $\hat{X}$. For example, it cannot be that $\hat{X} = 1$. With zero money growth, a firm could set a sticky price as an optimal markup over constant current and future marginal cost, and earn gross profits identical to those under flexible prices. Since this saves on the fixed cost, any firm would deviate to being sticky. For rule (10) to constitute a MPE policy rule, $\hat{X}$ must be large enough to ensure that the $\phi_i = \phi_{\text{max}}$ firm continues to choose flexibility.

The MA’s indifference at full flexibility also allows for mixed strategy policy rules. For instance, it is possible that when $z = 1$, the MA generates positive money growth, $\hat{X}_\delta > 1$, with probability $\delta < 1$, and zero money growth otherwise. For this to constitute a MPE, the mixing probability must be sufficiently large (see Appendix C). Let $\delta^{\text{min}}$ denote the smallest feasible mixing probability. Then for each $\delta \geq \delta^{\text{min}}$, there is a smallest feasible money growth rate – call this $\hat{X}^{\min}_\delta$ – ensuring that no firm deviates from flexibility to stickiness. The exact characterizations of $\delta^{\text{min}}$ and $\hat{X}^{\min}_\delta$ are in Appendix C. Here I summarize as follows:

**Proposition 5** Let $\delta \in [\delta^{\text{min}}, 1]$, where $\delta^{\text{min}}$ is defined in (15). Then for $\hat{X}_\delta \in \left[\hat{X}^{\min}_\delta, \infty\right)$, the discontinuous, mixed strategy policy rule (14) is a MPE policy rule.

This makes it clear that pessimistic equilibria can differ drastically across exogenous and endogenous price rigidity models. With exogenous price rigidity, pessimism is reflected in a unique, high value of equilibrium inflation. But with endogenous rigidity, a continuum of inflation rates can occur. Finally, it is worth noting that across all of these pessimistic MPE, real outcomes are identical to the case with a differentiable policy rule. The only difference across equilibria is in inflation rates.
7. A MODEL WITH UNIQUE EQUILIBRIUM

In the simplified model presented above, monetary policy affects real variables only when some prices are sticky. When prices are fully flexible, the MA is indifferent between all values of $X$. Here, I consider an arbitrarily small perturbation to the model to break the MA’s indifference. I introduce a non-zero cost of inflation that is present even when prices are fully flexible. This implies that equilibrium cannot exist with full flexibility.

In particular, suppose there is an arbitrarily small resource cost of money creation, $g = \varepsilon |X - 1|, \varepsilon > 0$. That is, printing (or shredding) money is costly in terms of final goods. The MA finances money creation via lump-sum taxation, so that the MA’s budget constraint is:

$$T_t = M_t - M_{t-1} - \theta_{t-1} \left( W_{t-1} h_{t-1} + \int_0^1 \Pi_{t,t-1} dt \right) - P_{t-1} g_{t-1}, \quad \forall t.$$ 

The modified model’s aggregate resource constraint is now:

$$c + g = y.$$ 

The rest of the model description is identical to Section 2. Apart from its effect via sticky prices, $X$ has a direct effect on the fraction of output available for consumption. So when prices are fully flexible, the maximizing MA sets $X = 1$ in order to minimize printing costs.

This strict preference for zero money growth at full flexibility introduces an obvious deviation for firms choosing flexibility. Suppose $z = 1$ so that $X = 1$. Given zero money growth, an individual firm considering a deviation to stickiness would set a sticky price as a markup over constant current and future marginal cost, $\tilde{p}' = \tilde{p} = \lambda \psi$. The firm would earn identical gross profits by choosing stickiness relative to choosing flexibility, but without incurring the fixed cost. As a result, any firm would deviate to stickiness, meaning that $z \neq 1$. No equilibrium exists with fully flexible prices.

The analysis of Section 6 indicates that in the unmodified model, for quantitatively reasonable specifications of the fixed cost of price change, prices are fully flexible in the pessimistic MPE. Hence, allowing for a cost of inflation independent of rigid prices implies that pessimistic equilibria do not exist. This cost need only be arbitrarily small. For
reasonable quantitative specifications, the modified model predicts a unique low inflation MPE, so there is no multiplicity.

7.1. Discussion

There are many ways to introduce a cost of inflation that is present with fully flexible prices, without changing the nature of the analysis. I discuss two possibilities. First, the cash-in-advance model considered here adopts Lucas’ (1982) timing of events within a period. Hence, any ‘money demand’ distortion is due to expected future inflation, which cannot be influenced by the MA under discretion. But in a model with Svensson’s (1985) timing, realized current inflation is costly since households use previously accumulated cash to conduct transactions. This type of portfolio rigidity is unrelated to price rigidity. Hence, Svensson’s timing would make current inflation costly even with flexible prices (for analysis of this cost in the context of monetary discretion, see Albañesi et al., 2001 and 2003).

A second way to modify the model is to change the timing of pricing decisions to more closely resemble a state-dependent pricing (SDP) model. In the model of Section 2, firms make their pricing decision before the realization of future inflation, i.e. in the first period of the two-period Taylor contract. In a SDP interpretation, the decision is made after observing future inflation, i.e. in the second period.26 Given perfect foresight, firms in the SDP version know ex-ante whether they will be changing their price ex-post and set first period prices accordingly. Hence, analysis of the two versions is virtually identical as long as \( z < 1 \).27 But a critical difference arises when \( z = 1 \), precisely because the firm’s decision to incur the fixed cost is made ex-post. Now, the MA’s choice of money growth

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26 See Devereux and Siu (2005). Though more familiar to the sticky price literature, the SDP version is notationally burdensome, complicating exposition. One needs to define an additional variable, \( \kappa' \), which measures the current period’s expected fraction of firms that choose price flexibility in the following period. Equilibrium requires \( \kappa' = z' \). Details are available upon request.

27 The only difference is that in making its policy choice, the MA accounts for the cost of labor resources devoted to price change (in addition to the cost associated with relative price distortions) when realized inflation is positive. Since the cost of price change is small (for plausible specifications of \( \phi_{\max} \)), this difference is quantitatively unimportant.
has a direct impact on the fraction of labor resources devoted to price change. Suppose all firms expect high inflation and set first period prices anticipating that they will be resetting them in the next period. If all firms are resetting prices, the MA’s influence via rigid prices is inoperative; the MA’s optimal choice is then to eliminate labor costs of price change by choosing $X = 1$. Clearly, this is susceptible to the same deviation on the part of firms as before. Expectations of high inflation with full price flexibility cannot be validated, so that self-fulfilling pessimistic MPE cannot exist for plausible fixed costs.

8. CONCLUSION

In this paper I have characterized time consistent equilibrium in a model with monetary discretion and an endogenously determined degree of price rigidity. The endogeneity is introduced by allowing firms to determine their frequency of price change; more frequent price change involves incurring a fixed cost.

When welfare costs of realized inflation are present only with sticky prices, there exist two time consistent equilibria: an optimistic equilibrium with low inflation, and a pessimistic equilibrium with high inflation. This is in keeping with previous studies which assume exogenously rigid prices. But for quantitatively reasonable specifications of the fixed cost, the pessimistic equilibrium displays full price flexibility. When an arbitrarily small cost of inflation exists independent of rigid prices, the pessimistic equilibrium is eliminated, and time consistent equilibrium is unique.

This suggests that the fragility of expectation traps relying on sticky prices is not limited to the framework studied here. As an example, consider a more elaborate specification of endogenous price rigidity, such as the state-dependent pricing (SDP) model of Dotsey et al. (1999). In both environments, the fraction of flexible prices is endogenous; however, with SDP, the number of past prices or ‘vintages’ inherited at any point in time is endogenous, while in the current two-period Taylor model, the number of past vintages is assumed to be one. Crucially, this paper finds that in high inflation settings, all firms prefer to set flexible prices as opposed to maintaining a sticky price that has been eroded by only one period.
of inflation. Clearly, no firm would prefer to charge a price eroded by multiple periods of inflation, so allowing for SDP would not impact on the analysis.

Finally, though attention has been restricted to Markov perfect equilibrium, the nature of the results is likely to extend to environments in which the monetary authority’s reputation matters. This is because the fragility of pessimistic equilibrium is due to optimizing behavior of private sector agents. Any proposed history of events that entails expectations of high inflation will result in firms opting for flexible as opposed to sticky prices. Again, this eliminates the monetary authority’s welfare trade-off in inflation due to price rigidity, leading to the arguments considered here.\(^{28}\) Simply put, it seems problematic to formulate an explanation for high inflation equilibria based on sticky prices since, in quantitative models, firms choose not to charge sticky prices.

**APPENDIX A**

Figures 3 through 5 are constructed as follows. Given a policy rule, \(\chi\), I calculate the steady state corresponding to the PSE when \(X = \chi (\bar{p})\) for a given value of \(\zeta\). Fix a value of \(\zeta\). The PS state is set to the corresponding steady state value, \((\bar{p}, z, X) = (\bar{p}_{ss}, z_{ss}, \chi (\bar{p}_{ss}))\). I then consider a range of prices for firms setting a two-period price in the current period. For each of these sticky prices, \(\bar{p}_j\), the best response, \(\bar{p}_i\), is given by:

\[\bar{p}_i = f (\bar{p}_j, z'; \bar{p}, z, X; \chi) = \lambda \psi \left[ (1 - \gamma (\bar{p}_j, z'; \bar{p}, z, X; \chi)) + \gamma (\bar{p}_j, z'; \bar{p}, z, X; \chi) \chi (\bar{p}_j) \right].\]

The relative weight on current versus future marginal cost must account for the fact that some firms choose flexibility. That is:

\[\gamma (\bar{p}_j, z'; \bar{p}, z, X; \chi) = \frac{p^{\lambda - 1}}{p^{\lambda - 1} + \beta p^{\lambda - 1} \chi (\bar{p}_j)^{\lambda - 1}},\]

\(^{28}\)An interesting open question is the characterization of sustainable equilibrium with endogenous price rigidity. In particular, the possibility that prices cease to be sticky for sufficiently high inflation can limit the severity of the worst sustainable equilibrium. Hence, the conditions under which first-best monetary policies can be sustained by trigger strategies may differ under endogenous and exogenous price rigidity.
where

\[ p^{\lambda-1} = \left\{ \frac{1}{2} \left[ (1 - z) \left( \frac{\tilde{p}}{X} \right)^{1-\lambda} + (1 - z') \tilde{p}'^{1-\lambda} + (z + z') \tilde{p}^{1-\lambda} \right] \right\}^{-1}, \]

\[ p'^{\lambda-1} = \left\{ \frac{1}{2} \left[ (1 - z') \left( \frac{\tilde{p}'_j}{\chi(\tilde{p}'_j)} \right)^{1-\lambda} + (1 - z'') \tilde{p}''^{1-\lambda} + (z' + z'') \tilde{p}'^{1-\lambda} \right] \right\}^{-1}. \]

Here, \( \tilde{p} = \tilde{p}' = \hat{\lambda}\psi, \) and \( \tilde{p}'' = P \left( \tilde{p}'_j, z', \chi(\tilde{p}'_j); \chi, \zeta \right) \) and \( z'' = Z \left( \tilde{p}'_j, z', \chi(\tilde{p}'_j); \chi, \zeta \right) \) are derived from the PSE decision rules. Determining \( \tilde{p}'' \) and \( z'' \) requires calculating \( \tilde{p}''', z''' \), and so on; these are also derived according to \( P \) and \( Z \).

For each \( \tilde{p}'_j \), I find \( z' \) as the value that satisfies:

\[
\frac{p^{\lambda-1}}{\tilde{p}^{\lambda}} (\tilde{p} - \psi) + \beta \left[ \frac{p'^{\lambda-1}}{\tilde{p}'^{\lambda}} (\tilde{p}' - \psi) - \psi F^{-1} (z') \right] = \]

\[
\frac{p^{\lambda-1}}{\tilde{p}'_j} (\tilde{p}'_j - \psi) + \beta p'^{\lambda-1} \left( \frac{\chi(\tilde{p}'_j)}{\tilde{p}'_j} \right)^{\lambda} \left( \frac{\tilde{p}'_j}{\chi(\tilde{p}'_j)} - \psi \right). \]

This is plotted in the bottom row of Figure 5. Using this value of \( z' \), I calculate \( \gamma \), and the best response price, \( \tilde{p}'_j \). In Figures 3 and 4, \( z = 0 \), and determining \( z' \) is not necessary.

**APPENDIX B**

The solution algorithm builds on a modified version of the MPE definition of Section 5. This modification is discussed in Klein et al. (2004), Subsection 3.2. Consider the following statement of the MA’s problem:

\[
\max_{p', z', X} \left[ U (s, X; \chi, \zeta) + \beta V (\tilde{p}', z'; \zeta) \right], \tag{11}
\]

subject to

\[
\mathcal{M} \equiv \left( p^{\lambda-1} + \beta p'^{\lambda-1} X'^{\lambda-1} \right) \tilde{p}' - \hat{\lambda}\psi \left( p^{\lambda-1} + \beta p'^{\lambda-1} X'^{\lambda} \right) = 0,
\]

\[
\mathcal{N} \equiv \left( p^{\lambda-1} + \beta p'^{\lambda-1} \right) \frac{\tilde{p} - \psi}{\tilde{p}^{\lambda}} - \beta \psi F^{-1} (z') - \left[ p^{\lambda-1} (\tilde{p}' - \psi) + \beta p'^{\lambda-1} X'^{\lambda} \left( \frac{\tilde{p}'}{X'} - \psi \right) \right] \frac{1}{\tilde{p}^{\lambda}} \geq 0,
\]

32
given $\zeta$, for all $s = (\bar{p}, z) \in \sigma$, with $N = 0$ whenever $F^{-1}(z') < \varphi_{\text{max}}$. Here, $p = p(s, X; \chi, \zeta)$ and $p' = p(s', X'; \chi, \zeta)$ are the current and future normalized price levels given by the pricing equation (1), $X' = \chi(s')$, $s' = (\bar{p}', z')$, and $\bar{p} = \hat{\lambda}_\psi$. Finally, $\bar{p}'' = \bar{P}(s'; \chi, \zeta)$ and $z'' = \bar{Z}(s'; \chi, \zeta)$ are one-period ahead pricing decisions taking as given that future money growth is determined according to $\chi(s')$; that is, $\bar{P}(s; \chi, \zeta) \equiv P(s, \chi(s); \chi, \zeta)$ and $\bar{Z}(s; \chi, \zeta) \equiv Z(s, \chi(s); \chi, \zeta)$. MPE requires that, given $\zeta$, the maximizing value of $X$ for all $s$ coincides with $\chi(s; \zeta)$. Furthermore, the solution to problem (11) coincides with $V(s; \zeta)$. This formulation represents a computational saving relative to Definition 3 as the dimension of the pricing decisions is reduced by one.

The GEE for problem (11) can be derived as follows. Let $\mu$ be the Lagrange multiplier associated with constraint $M$, and $\nu$ be the multiplier on constraint $N$. At an interior solution, constraints $M$ and $N$ must be satisfied with equality, and the FONCs can be rearranged to get:

$$U_c X + U_h X + \mu M X + \nu N X = 0, \quad (12)$$

where

$$\mu = - \frac{U_c p' + U_h p' + \beta V'_p - N_p \left(U_c c_{z'} + U_h h_{z'} + \beta V'_z\right) / N_z'}{M_p' - N_p' M_{z'}/N_{z'}},$$

$$\nu = - \left(U_c c_{z'} + U_h h_{z'} + \beta V'_z + \mu M_{z'}\right) / N_{z'}.$$

Equation (12) is the GEE. This expression depends on derivatives of the value function via the multipliers, as well as derivatives of the decision rules via the derivatives of the constraints. For example:

$$M_{z'} = (\lambda - 1) \left[p^{\lambda - 2} p_{z'} + \beta p'^{\lambda - 2} X'^{\lambda - 1} \Omega + \beta p'^{\lambda - 1} X' \lambda - 1 X'_{z'}\right] - \hat{\lambda}\psi (\lambda - 1) \left[p^{\lambda - 2} p_{z'} + \beta p'^{\lambda - 2} X' \lambda \Omega + \hat{\lambda}\beta p^{\lambda - 1} X' \lambda - 1 X'_{z'}\right],$$

where

$$\Omega = p'_z + p'_{X} \lambda' z' + p'_{\bar{p}'} \bar{P}' + p'_{z'} \bar{Z}'_z,$$

and similarly for $M_{p'}$, $N_{z'}$, and $N_{p'}$.

The following iterative algorithm makes use of the GEE to solve for MPE. The method begins with an initial guess of the policy rule, $\chi^0(\bar{p}, z; \zeta)$. I specify this guess as a tensor
product of Chebychev polynomials:

\[ \chi^0(\bar{p}, z; \zeta) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} a_{ij} T_i(\xi(\bar{p})) T_j(\vartheta(z)), \tag{13} \]

where \( T_i \) is the \( i \)-th order Chebychev polynomial, \( \xi \) is a linear function mapping a capture region of \( \bar{p} \), denote this \([\bar{p}_a, \bar{p}_b]\), into the interval \([-1,+1]\), and \( \vartheta \) maps \([0,1]\) into \([-1,+1]\).

The size of the approximation function is given by \( N \). An initial guess of the policy rule amounts to an initial guess on the coefficient vector \( \{a_{ij}\}_{i,j=0}^{N-1} \). Note that \( \chi^0 \) is differentiable and satisfies the limit condition of Subsection 6.1 by construction. Starting with \( i=0 \):

**Step 1.** Using \( \chi^i \), solve for approximations to the decision rules, \( \tilde{P} \) and \( \tilde{Z} \), over a grid of \( (\bar{p}, z) \) values, \( G = [\bar{p}_a, \bar{p}_b] \times [0,1] \), for the given \( \zeta \). This is done by: (a) specifying the decision rules to be of the same functional form as (13), (b) making an initial guess on the coefficient vectors, (c) solving constraints \( \mathcal{M} \) and \( \mathcal{N} \) for \( \bar{p}' \) and \( z' \) over \( G \), and (d) using the \( \bar{p}' \) and \( z' \) solutions to iterate on the coefficient vectors until convergence.

Note that \( \tilde{P} \) and \( \tilde{Z} \) are differentiable by construction. Using \( \chi^i, \tilde{P}, \) and \( \tilde{Z}, \) solve for \( \bar{p}' \) and \( z' \) values at \( z = 1 \); call these \( \bar{p}'_1 \) and \( z'_1 \).

**Step 2.** Using \( \chi^i, \tilde{P}, \tilde{Z}, \bar{p}'_1, \) and \( z'_1 \), solve for approximations to the value function, \( V(s; \zeta) \), \( s \in G \). This is done by: (a) computing the present discounted value of utility for each gridpoint, and (b) fitting a function of the form (13) to these points. Note that \( V \) is differentiable by construction. Also solve for \( V \) at \( z = 1 \); call this \( v_1 \).

**Step 3.** Using \( \chi^i, \tilde{P}, \tilde{Z}, \bar{p}'_1, z'_1, V, \) and \( v_1 \), solve the MA’s problem (11), given \( \zeta \), for \( X, \bar{p}' \), and \( z' \) over the grid \( G \). Whenever the solution is interior \( (z < 1) \), it can be found by satisfying constraints \( \mathcal{M} \) and \( \mathcal{N} \), and the GEE (12) with equality. When the solution is at a corner \( (z = 1) \), it can be found by performing a more tedious line-search for the maximizing value of \( X \). Use the \( X, \bar{p}' \), and \( z' \) solutions to get a new guess of the policy rule, \( \chi^{i+1} \).

Iterate on steps 1 to 3 until the coefficients on the policy rule converge. To check that the MPE policy rule is unique for a given \( \zeta \), do this for several initial guesses, \( \chi^0 \). To ensure that the policy rule is proportional in \( \bar{p} \), choose initial guesses that are non-linear in \( \bar{p} \).
C.1. Derivations for the Discontinuous Policy Rule

Here, I characterize \( \hat{X} \), the smallest admissible money growth at \( z = 1 \), such that (10) is a MPE policy rule. Consider all values of \( s = (\bar{p}, z) \) such that \( z = Z(s, \chi(s); \chi, \zeta) = 1 \), where \( \chi \) is the differentiable MPE policy rule of Subsection 6.1; denote these states as \( \hat{\sigma} \subseteq \sigma \).

Given that \( X' = \hat{X} \), flexible price profits for the \( \phi_i = \phi_{\max} \) firm at state \( \hat{s} \in \hat{\sigma} \) are given by:

\[
\tilde{\Upsilon}(\hat{s}; \zeta) \equiv \frac{p^{\lambda-1}}{\hat{\bar{p}}^\lambda} (\bar{p} - \psi) + \beta \left[ \frac{p'^{\lambda-1}}{\bar{p}'^\lambda} (\bar{p}' - \psi) - \psi \phi_{\max} \right],
\]

where \( \bar{p} = \lambda \psi \),

\[
p^{\lambda-1} = \left\{ \frac{1}{2} \left[ (1 - z) \left( \frac{\bar{p}}{\chi(\bar{s})} \right)^{1-\lambda} + (z + 1) \bar{p}^{1-\lambda} \right] \right\}^{-1},
\]

\[
p'^{\lambda-1} = \left\{ \frac{1}{2} \left[ (1 - z'') \bar{p}'^{1-\lambda} + (1 + z'') \bar{p}'^{1-\lambda} \right] \right\}^{-1},
\]

and \( z'' = Z(., 1; .) \) and \( \bar{p}'' = P(., 1; .) \) are the pricing decisions given \( z' = 1 \). Given that \( z' = 1 \), the decision rules \( P \) and \( Z \) are independent of the values of \( \bar{p}' \) and \( X' \). I index \( \tilde{\Upsilon} \) by \( \hat{s} \) to emphasize that flexible price profits depend on \( (\bar{p}, z) \) via \( p^{\lambda-1} \). If the firm chooses to deviate by charging a sticky price, it earns profits:

\[
\tilde{\Upsilon}(\hat{s}, \hat{X}; \zeta) \equiv \frac{p^{\lambda-1}}{\bar{p}^\lambda} (\bar{p}' - \psi) + \beta \left( \frac{\hat{X}}{\bar{p}'} \right)^{\lambda} \left( \frac{\bar{p}'}{\bar{p}^\lambda} - \psi \right),
\]

where

\[
\bar{p}' = \hat{\lambda} \psi \left( \frac{p^{\lambda-1} + \beta p'^{\lambda-1} X^\lambda}{p^{\lambda-1} + \beta p'^{\lambda-1} X^\lambda} \right).
\]

In order for the deviation to be unprofitable, it must be that:

\[
\tilde{\Upsilon}(\hat{s}; \zeta) \geq \tilde{\Upsilon}(\hat{s}, \hat{X}; \zeta).
\]

Let \( \hat{X}(\hat{s}; \zeta)^{\min} \) denote the smallest \( \hat{X} \) such that this holds at \( \hat{s} \). This condition must hold for all \( \hat{s} \in \hat{\sigma} \). Hence, in order for \( \hat{\chi} \), with \( \hat{\chi}(\bar{p}, 1; \zeta) = \hat{X} \), to constitute a MPE policy rule it must be that \( \hat{X} \geq \hat{X}^{\min} \) where:

\[
\hat{X}^{\min} = \max_{\hat{s} \in \hat{\sigma}} \left\{ \hat{X}(\hat{s}; \zeta)^{\min} \right\}.
\]

Consider policy rules of the following form:

\[
\hat{\chi}_{\delta} (\bar{p}, z; \zeta) = \begin{cases} 
\chi (\bar{p}, z; \zeta) & \text{for all } \bar{p} \text{ and } z < 1 \\
\hat{X}_{\delta} & \text{for all } \bar{p} \text{ and } z = 1 \text{ with probability } \delta \\
1 & \text{for all } \bar{p} \text{ and } z = 1 \text{ with probability } 1 - \delta
\end{cases}, \quad (14)
\]

where \( \chi \) is the differentiable MPE policy rule of Subsection 6.1. When the MA inherits \( z = 1 \) it generates positive money growth, \( \hat{X}_{\delta} > 1 \), with probability \( \delta \), and zero money growth otherwise. Showing that rule (14) is a MPE policy rule entails checking that at \( z = 1 \), no firm deviates to stickiness.

Ruling out such deviations requires restricting the admissible values of \( \delta \). For instance, in the neighborhood of \( \delta = 0 \), the optimal sticky price implies negative profit when \( \hat{X}_{\delta} \) is realized. Given the option, the firm would choose to shut down rather than meet demand. Hence, for \( \delta \) sufficiently small, a sticky price firm will find it optimal to simply set a price in anticipation of zero money growth and shut down when positive money growth occurs. Accounting for shut down puts a lower bound on the set of feasible \( \delta \) values.

Let \( \delta_{\text{min}} \) denote the smallest admissible mixing probability such that (14) constitutes a MPE with the option of shut down. It suffices to check that the \( \phi_i = \phi_{\text{max}} \) firm does not deviate to stickiness. Denote the states such that \( z' = Z(s, \chi (s); \zeta) = 1 \) as \( \hat{\sigma} \subseteq \sigma \). Since the deviating firm shuts down when \( \hat{X}_{\delta} \) is realized, it is pricing only for the zero money growth state in the future and charges a price identical to the optimal flexible price, \( \bar{p}' = \bar{p} = \hat{\lambda} \psi \).

Profits from this deviation are:

\[
\Theta (\hat{s}; \zeta) \equiv \frac{p^{\lambda-1}}{\bar{p}^{\lambda}} (\bar{p} - \psi) + \beta (1 - \delta) \frac{p'^{\lambda-1}}{\bar{p}^{\lambda}} (\bar{p} - \psi),
\]

where \( p^{\lambda-1} \) and \( p'^{\lambda-1} \) are as given in Subsection C.1 with \( \hat{\chi} (\hat{s}; \zeta) \) replaced by \( \hat{\chi}_{\delta} (\hat{s}; \zeta) \), \( \hat{s} \in \hat{\sigma} \). To ensure that this is not profitable, it must be that \( \hat{\Upsilon} (\hat{s}; \zeta) \geq \Theta (\hat{s}; \zeta) \); simplifying this condition indicates that it holds whenever \( \delta \geq (\bar{p}^{\lambda} \phi_{\text{max}}) / \left[ p'^{\lambda-1} \left( \hat{\lambda} - 1 \right) \right] \). This condition is independent of \( \hat{s} \) and \( \hat{X}_{\delta} \). Hence, the smallest feasible mixing probability is:

\[
\delta_{\text{min}} = \frac{\bar{p}^{\lambda} \phi_{\text{max}}}{p'^{\lambda-1} \left( \hat{\lambda} - 1 \right)}. \quad (15)
\]
For all $\delta \geq \delta^{\text{min}}$, the value of $\hat{X}^{\text{min}}_{\delta}$ is defined in an identical fashion to $\hat{X}^{\text{min}}$ above.

As an illustration, Figure 7 plots $\hat{X}^{\text{min}}_{\delta}$ for $\delta \in [\delta^{\text{min}}, 1]$. The figure is plotted for the baseline calibration, with $\phi_{\text{max}} = 3.6\%$. As $\delta$ increases, the smallest feasible value of positive money growth, $\hat{X}^{\text{min}}_{\delta}$, at first falls and then increases. I briefly discuss the source of this non-monotonicity between $\delta$ and $\hat{X}^{\text{min}}_{\delta}$. Again, $\hat{X}^{\text{min}}_{\delta}$ is defined as the value of money growth such that $\tilde{\Upsilon}(\hat{s}; \hat{\zeta}) \geq \tilde{\Upsilon}(\hat{s}, \hat{X}^{\text{min}}_{\delta}; \hat{\zeta})$. Since, flexible price profits are independent of $\delta$ and $\hat{X}_{\delta}$, the slope $\partial \hat{X}^{\text{min}}_{\delta}/\partial \delta$ depends on the effect of $\delta$ on sticky price profits, $\partial \tilde{\Upsilon} / \partial \delta$, for a given $\hat{X}_{\delta}$. Since $\partial \tilde{\Upsilon} / \partial \hat{X}_{\delta} < 0$, $\displaystyle \left[ \partial \hat{X}^{\text{min}}_{\delta} / \partial \delta \right] = \text{sign} \left[ \partial \tilde{\Upsilon} / \partial \delta \right]$. The U-shaped pattern is due to the fact that a change in $\delta$ affects sticky price profits via two offsetting channels: first, through the change in weight placed on profits across the positive and zero inflation states; and second, through the change in profits earned in each state, due to the effect of $\delta$ on the optimal sticky price, $\hat{\rho}'$. Whether the slope, $\partial \hat{X}^{\text{min}}_{\delta} / \partial \delta$, is positive or negative depends on the sign of each effect and the relative strength of each effect at a particular $\delta$. Further details are available from the author upon request.

REFERENCES


MA chooses $X$ after observing $s = (\bar{p}, z)$

private sector makes decisions, including $(\bar{p}', z')$, after observing $(s, X)$

MA chooses $X'$ after observing $s' = (\bar{p}', z')$

Figure 1. Timing of events within a period.
Figure 2. Best response function: zero money growth.
Figure 3. Best response functions: linear policy rule. Solid line: future expectations coordinated on low inflation equilibrium; dashed line: future expectations coordinated on high inflation equilibrium.
Figure 4. Best response function: non-linear policy rule.
Figure 5. Best response function and fraction of firms choosing price flexibility. Left column: large maximal fixed cost. Right column: small maximal fixed cost.
Figure 6. Maximal fixed cost such that pessimistic MPE displays full price flexibility in steady state, for various values of $\lambda$. 

*some price rigidity in high inflation steady state*

*full price flexibility in high inflation steady state*
Figure 7. Minimum money growth rates when MA plays discontinuous, mixed strategy policy rule.